Load-Rating Based Bridge Deterioration Model

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Summary

In managing in-service bridges it is very important that the change in bridge performance over time be known. Existing bridge management systems (BMS), for example, Pontis [1] uses the relationship between condition rating and time. However, load rating is also a useful measure of performance. It is logical that the relationship of the load rating and time should also be useful in BMS applications. The authors had proposed a method based on survival analysis to study such relationship. The deterioration process is modeled as a semi-Markov process.

1. Introduction

Bridge deterioration models are an essential component of a computerized bridge management system (BMS). It allows prediction of future bridge performance and needs. Existing BMSs, for example, Pontis [1] model bridge deterioration as the decline of condition rating over time. Condition rating is an ordinal scale consisting of numerical values assigned to represent each condition state of the bridge or its components. If we define state 1 as the best state, state 2 as the next state, so on and so forth then a typical bridge deterioration model would look like a stepped function as in Fig. 1.

Condition rating, however, is not the only measure of performance. In the US, the theoretical load carrying capacity of a bridge is evaluated and expressed in respect to a standard vehicle as load rating. Load rating may indeed be a better indicator of performance level as compared to condition rating. Firstly, it can be objectively calculated rather than by subjective evaluation. Secondly, it is in a continuous scale rather than a discrete scale so it can assume any real value. Thirdly, it is a ratio scale rather than an ordinal scale and thus is more amenable to mathematical manipulation. In fact, load rating is concerned with a different aspect of measuring bridge performance. It measures not only the capacity of structural members but indeed the capacity in relation to live loads. This way, it permits use of site specific information in its evaluation. In many instances, for example, in posting weight restriction on bridges load rating is a better criterion for decision making. It is logical that the relationship of the load rating with time should also be useful in managing existing bridge stocks. We call this type of relationship a load-rating based bridge deterioration model.

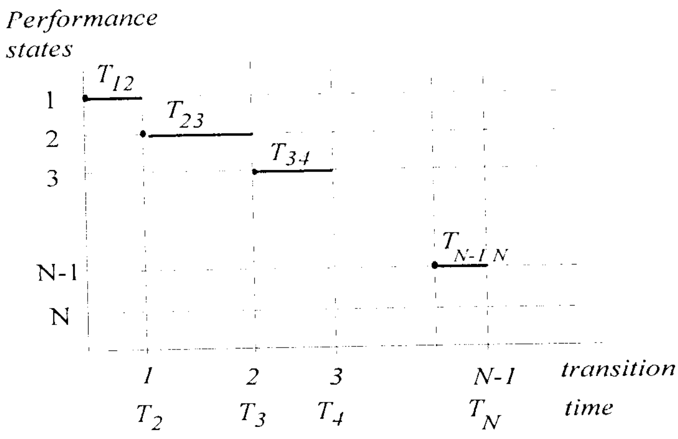

Fig. 1 Bridge Deterioration Process

2. The Modeling Procedure

A good discussion of bridge deterioration modeling can be found in [2,3]. Ref.[2] describes use of Markov model while ref.[3] considers a more general semi-Markov model. A semi-Markov process is a class of stochastic process that is governed by two different and independent random-generating mechanisms. When the process enters any state i, the probability of it moving to the next state j in the next transition is specified by the transition probability p_{ij}. Once the successor state is determined the process stays in the current state i for a duration T_{ij} which is dictated by the holding time probability density function $h_{ij}(m)$. In a bridge deterioration process the performance states would drop deterministically from one state to the next worse state; in a stepped function; as depicted in Fig. 1. Modeling of bridge deterioration process as a semi-Markov process therefore involves only the determination of the holding time distributions $h_{ij}(m)$.

Consider Fig. 1, if the probability distribution of the time to reach each state i (from state 1), viz., T_i is known then it may be possible to determine the distributions for holding times T_{ij} by taking the difference of T_j and T_i.

2.1 Time to state *i*, T_i

The time to reach a state T_i could be conceived as the time to failure or *failure time*. "failure" is used here to denote a distinct event, in this case, the reaching of the performance level represented by state i. The study of failure time has been the subject of a statistical analysis called survival analysis [4].

In traditional applications, *failure time* data is obtained from *life testing* (in industrial applications) or *clinical trials* (in medical applications). In either case, a specific number of the subjects of interest are observed for a period of time to obtain their individual times to failure (or time to the relapse of a certain decease). It is not uncommon to find that some of these items on test have been lost to follow up the study or have continued to survive at the end of the study period. As a result, the failure times of these subjects are not observed. The observations are said to be *censored*. Uncensored observations of failure times are called *complete* observations.

Bridges could not be subject to life testing for obvious reasons. A procedure proposed by Ng and Moses [3,5] use NBI (national bridge inventory) data for survival analysis. The data is treated as if obtained from a life test in which the addition of a new bridge in the inventory is regarded as an entry of the bridge in the life test. The ages of the bridge are regarded as an observed failure times. Whether an observed failure time is right-censored, complete or left-censored would depend on whether the reported performance is less than, equal or more than the specified state; respectively. By successively specifying the performance states $i = 2, 3, \ldots, N$ the time to reach each state (from state 1) can be derived using the procedure discussed above.

2.2 Holding time

Next is to determine the holding time distributions from knowledge of the distributions of T_i and T_j. Consider the difference of two arbitrary random variables $Z = X - Y$, the object now is to derive an expression for the random variable Z from the PDFs of X and Y. In particular, consider that Y is Weibull distributed with parameters (α_1, κ_1) and X is Weibull distributed with parameters (α_2, κ_2). Ng & Moses [3] assumes that the number of transitions taking place in the deterioration process is governed by a nonhomogeneous poisson process called quasi-Weibull process. This process is a naïve adaptation of a Weibull process. Some discussion of Weibull process is given in [6]. Based on this assumption the PDFs of Z can be obtained from Eq. 1.

$$f_Z(z) = \int_z^{\infty} \frac{\alpha_1\alpha_2\kappa_1\kappa_2 (x-z)^{\kappa_1-1} x^{\kappa_2-1} \exp\left[-\alpha_1 (x-z)^{\kappa_1}\right]}{\exp\left[-\alpha_2 (x-z)^{\kappa_2}\right]} \cdot \exp[-\alpha_2 x^{\kappa_2}]dx$$

$$= \alpha_1\alpha_2\kappa_1\kappa_2 \int_z^{\infty} (x-z)^{\kappa_1-1} x^{\kappa_2-1} \cdot \exp\left[-\alpha_1 (x-z)^{\kappa_1} - \alpha_2[x^{\kappa_2} - (x-z)^{\kappa_2}]\right]dx \qquad (1)$$

By letting $Y = T_i$ and $X = T_{i+1}$, the PDFs of the holding time, $h_{i+1}(m)$ for $i = 1, 2, \ldots N$ can be computed using Eq. (1).

3. Connecticut Model

The NBI data (1989-1993) from Connecticut, U. S. A. was used to illustrate the procedures described above. The bridges were divided into their respective groups in terms of their construction materials and design load. A separate model was developed for each group of bridges. Steel bridges designed to HS20 (Group C) constitute majority of the bridge stock, i.e., 81.13% and will be reported here.

Load rating represents the load-carrying capacity of a bridge in terms of the gross weight (in tons) of standard vehicles measured in a continuous scale. To use the semi-Markov representation the scale is arbitrarily discretized into 11 states as in Table 1. For more exact modeling of the continuous scale more states could of course be used.

Table 1
Performance States Defined

Performance state, ξ	Load rating at opertaing level (tons)	Performance state, ξ	Load rating at opertaing level (tons)	Performance state, ξ	Load rating at opertaing level (tons)
1	> 97.5*	5	(67.5, 72.5]	9	(47.5, 52.5]
2	(82.5, 97.5]	6	(62.5, 67.5]	10	(42.5, 47.5]
3	(77.5, 82.5]	7	(57.5, 62.5]	11	< 42.5
4	(72.5, 77.5]	8	(52.5, 57.5]		

4. Results

Survival analysis essentially fit the failure time data to a probability distribution; very often, in the form of survival function. A survival function is defined as $\Pr\{T > t\}$, that is, the time function of the probability that the time to failure T exceeds time t. In this example, the data was fit with Weibull distribution and the goodness of fit was found to be good. SAS [7] procedure *lifereg* was used to estimate the Weibull parameters and the results obtained were presented in Table 2.

Tables 2
Results of Survival Analysis

Load	*Group C: steel, design load = HS20*	
Rating (tons)	Scale parameter α	Shape parameter κ
(82.5, 87.5]	0.4464713	0.67125627
(77.5, 82.5]	0.26666335	0.71838668
(72.5, 77.5]	0.14258163	0.82704777
(67.5, 72.5]	0.04877082	1.06634705
(62.5, 67.5]	0.01074504	1.38615893
(57.5, 62.5]	0.00235123	1.59552996
(52.5, 57.5]	0.00133146	1.4472555
(47.5, 52.5]	0.00061842	1.44524961
(42.5, 47.5]	5.46E-05	1.92116315
<42.5	4.62E-07	3.06357531

The parameters in Table 2 were used for plotting the survival functions as presented in Fig. 2.

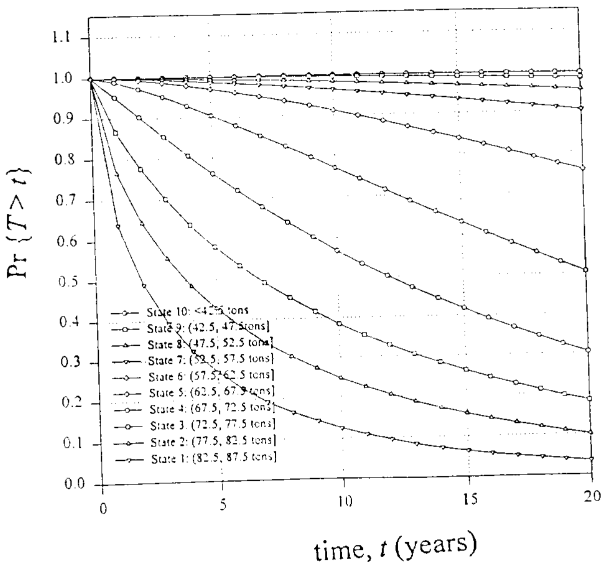

Fig. 2 Survival Functions

* Not including 99 because the entry of '99' in the NBIcoding represents "rating not available."

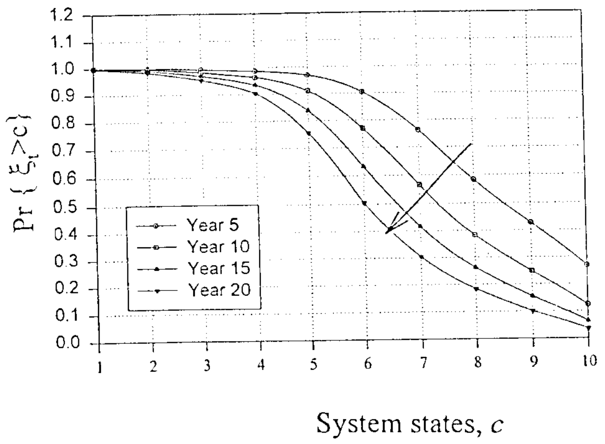

Fig. 3 Complementary cdf of Load Rating

The family of survival functions in Fig. 2 together contain much information about the deterioration process of Group C bridges. Each survival function represents the probability distribution of the time to reach a certain performance state. Eq. (1) is then used to determine the holding time distributions as discussed in Section 2.

It would be interesting to examine the survival functions from another angle. If we were to cut across the family of survival functions in Fig. 2 at any time t we would obtain a function which is the cumulative distribution function (cdf) for random performance state. The performance state at time t = 5, 10, 15 and 20 years are separately treated here as a random variable whose complementary cdf's are plotted in Fig. 3. The graphs exhibits a shift of the performance state distribution due to the passage of time from year 5 to year 20.

5. Applications

One major application of a bridge deterioration model is the prediction of future performance. For performance prediction (under no human intervention), Eq. (2) is applicable [2].

$$\pi(m) = \pi(0) \times \{ \psi_{ij}(m) \} \tag{2}$$

where $\pi(m)$ is the state probability vector at time m and$\{ \psi_{ij}(m) \}$is the *interval probability matrix.* The element $\psi_{ij}(m)$ is the probability that the process will occupy state j at time n given that it entered state i at time zero. Consider m as the time for the first transition to occur after the process entered state i, ref. [2] gives two equations for the determination of $\psi_{ij}(m)$. For $m < n$, the interval probability is given in Eq. (3). For $m > n$, Eq. 4 applies.

$$\psi_{ij}(m) = \sum^{n}_{m=1} h_{ii+1}(m)\ \psi_{ij}(n\text{-}m),\ j \neq i \tag{3}$$

$$\psi_{ij}(m) = 1 - {}^{\leq}h_{i\ i+1}(m),\quad j \neq i \tag{4}$$

where the notation ${}^{\leq}h_{i\ i+1}(m)$ is used to denote the cumulative distribution function of the holding time $h_{i\ i+1}(m)$.

Using Eq. (3) and Eq. (4) the interval probability for each i-j pair was computed using a computer software called *Mathematica* [8]. Only the matrix for m = 10 years are presented.

Table 3 Interval Probability Matrix, $\Psi(m)$

$\Psi(m) = \{ \psi_{ij}(m) \}, i = 1, 2, ..., 11; j = 1, 2, ..., 11$

m=10

0.203	0.418	0.279	0.086	0.012	0.001	0.000	0.000	0.000	0.000	0.000
0.000	0.399	0.394	0.171	0.033	0.003	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.478	0.386	0.120	0.015	0.001	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.550	0.366	0.078	0.005	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.637	0.323	0.039	0.001	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.769	0.221	0.010	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.904	0.094	0.002	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.945	0.054	0.001	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.950	0.046	0.004
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.817	0.183
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

6. Conclusions & Recommendations

This paper implements two new ideas in bridge deterioration modeling. They are:

i. Use of load rating in deterioration modeling;
ii. Use of semi-Markov formulation;

The use of load rating in the deterioration models allows bridge management decisions which are based on load rating. Examples are bridge posting, issuance of special permits for oversized vehicles. Also, the load rating-based deterioration model facilitates use of structural reliability theory in project-level bridge management decisions.

One possible extension of this model is the use of continuous scale for performance level instead of arbitrarily divide the scale in discrete states.

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