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PREDICTION OF BRIDGE SERVICE LIFE USING TIME-DEPENDENT RELIABILITY ANALYSIS

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Abstract

A bridge deterioration model is an essential component of a computerised bridge management system (BMS). Existing BMSs use Markov chain theory to model the deterioration process as a decay of condition ratings over time. An alternative approach based on time-dependent reliability theory is proposed. The new approach is in principle a generalisation of the Markov chain models. Rather than addressing the stochastic nature of condition rating the proposed approach seeks to model the random time using survival analysis.

Keywords: Bridge management system (BMS), Markov chain theory, time-dependent reliability, survival analysis, bridge deterioration models, bridge life prediction.

1 Introduction

In developing a computerised bridge management system (BMS) there is need to model the bridge deterioration process. The idea is: if we knew the rate and/or pattern of bridge deterioration we would be able to predict the remaining lives as well as future performance of the bridges in the network. Both of these are important inputs to the decision models of a bridge management system.

The bridge deterioration process has often been modelled as the decay of bridge performance over time. Earlier models tend to be simple with linear and deterministic deterioration curves. More recent models used in the Indiana State's BMS [1] and FHWA-sponsored BMS software *Pontis* [2] are based on Markov chain theory. The deterioration process, recognized as a stochastic process, is represented by the transition probability matrix:

$$\mathbf{P} = \{p_{ij}\} \quad (1)$$

where p_{ij} is the probability of the bridge transiting from state i to state j in one step. A stochastic process $\{Z_n: n=0, 1, \dots\}$ is defined as an indexed set of random variables, Z_n . It depicts the value of the system state Z_n at each time point, n . In the Markov-chain bridge deterioration model, the system states are indicated by some ratings of bridge condition.

Existing bridge deterioration models based on the Markov chain theory invariably assumes that a bridge can either remain in the current state or deteriorate to the next state in one transition. Also, the worst state M in a state space of $\{1, 2, \dots, M\}$ is considered an absorbing state; which means that once the process enters the state it will never leave it. The stochastic nature of the deterioration process is thus described by the transition matrix of this format:

$$\mathbf{P} = \begin{bmatrix} p_1 & 1-p_1 & \cdots & 0 & 0 \\ 0 & p_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{M-1} & 1-p_{M-1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (2)$$

where p_i , $i=1, 2, \dots, M-1$ represents the probability of remaining in the i th state in the next transition. Notice that p_M is equated to 1 since M is an absorbing state.

The expected condition of the bridge at a future time n or conversely, the expected time to reach any specific future state can be calculated by using this relationship:

$$\pi^{(n)} = \pi^{(0)} \times \mathbf{P}^n \quad (3)$$

where $\pi^{(n)}$ is the state probability vector at any time n and $\pi^{(0)}$ is the initial state probability vector. The stochastic process is thus fully specified once the transition matrix \mathbf{P} as well as the initial states are known.

There is wide acceptance in the use of Markov chain theory in deterioration modelling but three issues need further investigation:

1. Suitability of the Markov chain theory in bridge deterioration modelling
2. Suitability of condition rating as the bridge performance measure
3. Methods of estimating the transition probabilities, p_{ij}

We briefly discuss these issues in section 2 to provide the background for introducing an alternative approach to model bridge deterioration process. It is later shown that the proposed method based on time-dependent reliability theory amounts to the generalisation of the Markov chain model.

2 The state of the art

2.1 Markov chain theory

Markov chain theory is founded on two fundamental rules: *memoryless* and *homogeneous*. The memoryless rule stipulates that the future states of the process depend only on the current states; while the homogeneous rule requires that the rates of transition from one state to another remain constant throughout the time. Translated to the Markov-chain bridge deterioration model this is saying that the transition probabilities depend only on the current states and not on the ages of the bridges. Thus, a 5-year old bridge and an 80-year old one, should they be of the same ratings, are equally likely to stay in their current state in the next transition. *Pontis* [2] assumes this to be valid and uses only one transition matrix for the whole life span.

Indiana BMS develops separate transition matrices for each age group [1]. In this way, the stochastic nature of the deterioration process depends on both the current states as well as the ages of the bridges. One disadvantage of this effort however, is that by zoning there is a much smaller sample size within each group which reduces the precision of the estimators.

2.2 Performance measure/ state

Almost all existing bridge deterioration models use condition rating as the measure of bridge performance. Indiana uses the rating system introduced by the U. S. Federal Highway Administration (FHWA) [4]. Numeric ratings of 0 to 9 are used to indicate the physical conditions of bridge deck, superstructure and substructure; with 9 representing the "excellent condition" and 0 the "failed condition". *Pontis* [3] uses a different system of rating between 1 and 5. 1 is the 'best' and 5 is the 'worst' state.

It has been observed that condition rating is not adequate as a performance measure [2]. Condition rating does not reflect the structural integrity of a bridge; nor the improvement needs. Indeed, many major bridge management decisions, for example, posting have been based on the load rating. Besides, because condition rating is in the ordinal scale, we cannot compare two ratings by their difference or ratio. There is actually suggestion that bridge deterioration process modelling should include load rating [5].

2.3 Estimation of transition probabilities

Two methods of estimating the transition probabilities have been considered and discussed by Jiang et al [1] for the Indiana BMS. We will call them the 'Frequency' approach and the 'Regression' approach. In the Frequency approach, the transition probability p_{ij} is estimated by

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} \quad i, j = 1, 2, \dots, M \quad (4)$$

where n_{ij} is the number of bridges originally in state i which have moved to state j in one step; and n_i is the total number of bridges in state i before the transition. \hat{p}_{ij} has been shown to be a maximum likelihood estimator (MLE) [6]. From Eq.(4) it is

clear that this approach would require at least two sets of inspection data pertaining to two different points in time.

In the Regression approach, only one set of bridge data is needed. A regression function is first obtained by regressing condition ratings on ages. Transition probabilities are then estimated by "fitting" the regression function with the transition matrix. This involves seeking an optimal solution to minimize the difference between the expected condition rating (from the regression function) and that derived from the transition matrix.

The approach adopted by Pontis [2] for estimating the transition matrix is in reality the 'Frequency' approach. However, instead of relying on bridge data which is deemed to be scarce at the beginning of system implementation, the proportions of bridges to transit from one state to another are to be elicited from the bridge experts. As more data becomes available after subsequent inspections in the following years the probabilities are to be updated using Bayesian method.

3 Reliability-based performance measure

3.1 Time-dependent reliability

In previous sections we have seen the treatment of condition ratings as the response variable subject to influence of ages and other explanatory variables. Since the ratings are random variables, it makes sense to talk about the probability of these ratings reaching or exceeding a certain threshold value within the time interval $[0, t]$. We can indeed establish the relationship of this probability (or rather, its complement we called reliability) with time. This is the basis of time-dependent reliability theory.

We define the reliability function $S(t)$ as the probability of survival of a system within the time $[0, t]$. In other words, it is the probability that the time to failure exceeds the time, t :

$$S(t) = P[T > t] \quad t \geq 0 \quad (5)$$

T is a non-negative random variable representing the time to failure and is commonly known as the failure time or lifetime. 'Failure' in this context refers to the event that the state hits a well-defined threshold value for the first time.

The reliability function explicitly expresses the reliability of a *new* bridge at any point in time. For an in-service bridge we would use an equivalent function known as *hazard function*, $h(t)$. The hazard function specifies the instantaneous rate of failure at time t given that the individual survives up till time t . It can be proven that

$$h(t) = f(t) / S(t) \quad (6)$$

where $f(t)$ is the probability density function of T . Given the distribution of T in any of these forms, information about the remaining life and future bridge performance can be determined. As an example, the mean residual life can be calculated using

$$m(t) = E[T-t|T>t] \quad (7)$$

The question now is how are we to estimate the lifetime distribution.

3.2 Estimation of the bridge reliability function

Estimation of the reliability function from lifetime data is the subject matter of *survival analysis* commonly used in the industry for reliability testing of machines; and in biomedical fields for prediction of life expectancy. It is indeed a regression analysis of lifetime T (rather than the condition rating) on the explanatory variables (called 'covariates' in survival analysis). Also, it fits a distribution function rather than the expected value to the field data. What is unique in this statistical technique is the presence of censored observations. Censored observations are not complete. If we know that a bridge reaches a certain well defined threshold value at an age y , we have a complete observation: y is the lifetime value. However, if we found at the time of bridge inspection that a bridge had not reached the limiting value, then we have a right-censored observation. This observation though incomplete is still useful for it tells us that the lifetime of the bridge goes beyond its present age. If instead we found that a bridge had already surpassed the limiting value at the time of inspection, we then have a left-censored observation. We know that the lifetime of the bridge is less than or equal to the present age.

Based on this concept and using the 1991 NBI data from the state of Indiana, a parametric fitting with Weibull distribution was carried out using SAS procedure 'lifereg' [7]. This procedure handles doubly censored observations and includes checks on the significance of the estimated parameters. The threshold value for 'failure' was specified at the condition rating of '3'. This was to follow Jiang et al [1] so a comparison with their results could be made. Three covariates had been considered: material type, average daily traffic (ADT) and a categorical variable to indicate if the bridge has previously been rehabilitated. It was found that ADT does not significantly affect the time to failure and was thus dropped from the analysis.

The Weibull parameters were computed from the results of the analysis. For illustration, parameters corresponding to threshold value of '3' are given in Table 1 and related distributions plotted in Fig. 1. The curves suggest that steel bridges tend to take a shorter time to reach condition rating '3' as compared to concrete bridges. Also, rehabilitation work done on bridges do have positive effect in extending the bridge lives.

Table 1. Fitted parameters of Weibull distribution (threshold value=3)

	Conc. - rehab.	Conc. No rehab.	Steel - rehab.	Steel No rehab.
α , Scale param	1.43E-08	6.08E-09	4.08E-08	1.73E-08
γ , Shape param.	3.6315	3.6315	3.6315	3.6315

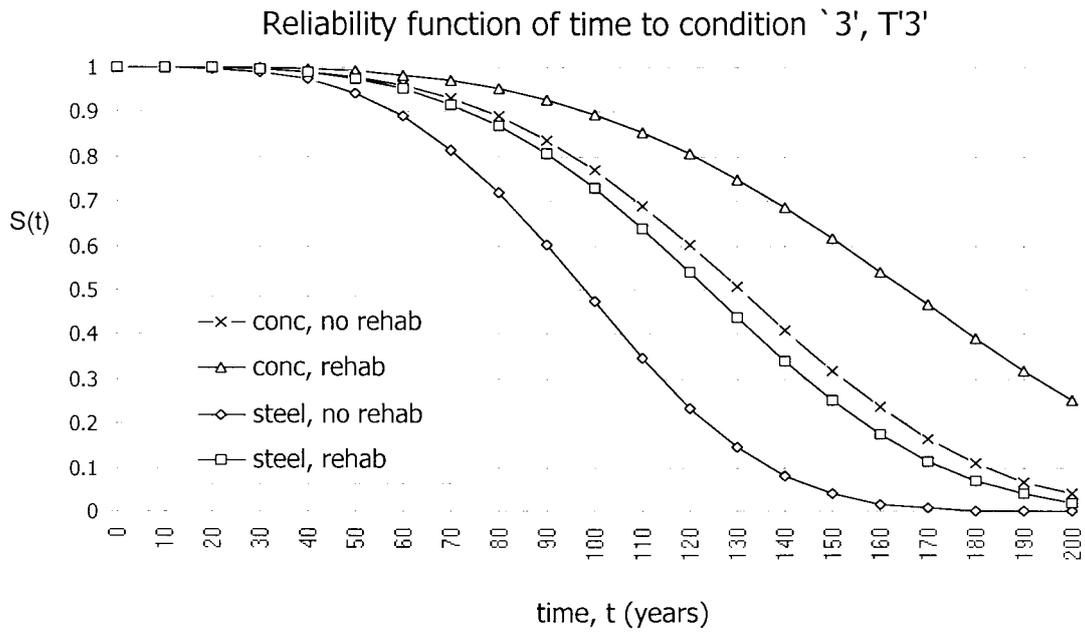


Fig. 1. Reliability functions of bridges fitted by Weibull distribution

A close examination of the reliability functions suggests that they may indeed be a plot of an infinite number of two-state Markov chains along the time axis. To investigate we arbitrarily divide the time axis into various disjoint intervals of unit length. If we denote '1' as survival and '2' as failure we then obtain, for each time interval, a two-state Markov chain:

$$P_m = \begin{bmatrix} p_m & 1-p_m \\ 0 & 1 \end{bmatrix} \quad (8)$$

where $1-p_m$ is the probability of failure in the interval $[m-1, m]$ and p_m is the probability of survival beyond the interval. We know that for a two-state Markov chain with the format as in (8) the unconditional probability of survival at time n in the future is given by

$$P_m = \begin{bmatrix} p_m & 1-p_m \\ 0 & 1 \end{bmatrix} \quad m = 1, 2, \dots \quad (9)$$

This is indeed the nonparametric estimator for $P[T>t]$; viz., the reliability function.

To further investigate the nature of the reliability function we proceeded to derive a homogeneous Markov chain from the survival model. By successively redefining the limiting value for 'failure', that is, '3', '4', ..., '8'; and performing the survival analysis in each case we obtained distributions of $T_{3'}$, $T_{4'}$, ..., $T_{8'}$. These distributions together give a complete description of the deterioration process; as illustrated in Fig. 2.

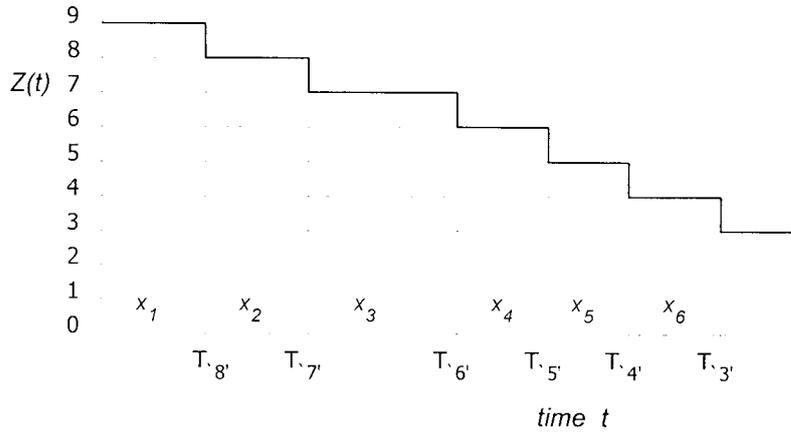


Fig. 2. A sample function of bridge deterioration process

The random variable X_i is the duration that the process takes to stay in state i and is sometimes called the *sojourn time*. Notice that if X_1, X_2, \dots , have independent and identical exponential distributions we would have a continuous-time Markov process. In the present case, they are neither exponential nor identical.

We next define $Q_{ij}(t)$ as the probability that after entering state i the process will next move to state j in an amount of time less than or equal to t . Put in another way, it is the probability that the random time for the process to move from i to j is less than or equal to t . In the special case where the bridge condition only deteriorates but never improves we have:

$$\begin{aligned}
 Q_{12}(t) &= P[X_1 \leq t] = P[T_{8'} \leq t] \\
 Q_{23}(t) &= P[X_2 \leq t] = P[T_{7'} - T_{8'} \leq t] \\
 &\vdots \\
 &\text{etc.}
 \end{aligned}
 \tag{10}$$

We know that T are Weibull distribution and Eq.(10) can be solved as a convolution:

$$P[T1 - T2 \leq t] = \int_0^t F_{T2}(t + \tau) f_{T1}(\tau) d\tau; \quad \text{for any } T1 \text{ and } T2.
 \tag{11}$$

We considered a transition period of 1 year and set $t = 1$. Transition probabilities were obtained by solving Eq.(11) numerically. Table 2 shows the transition matrix for concrete bridges which have not been rehabilitated previously.

4 Discussions & conclusions

This paper has discussed existing methods for estimating transition probabilities of Markov-chain bridge deterioration model. An alternative approach using time-dependent reliability and survival analyses has been proposed and some preliminary results shown. The advantages of the proposed method include:

- Use only one set of data

- Sojourn time is not restricted to geometric or exponential distribution
- Use of probability-based measure like reliability function facilitates computation of risk needed in life cycle cost analysis

However, like the Markov chain models discussed in this paper, the proposed model could be used only for decisions regarding a network of bridges rather than on individual bridges.

Table 2. Transition matrix for concrete bridges without previous rehabilitation

	9	8	7	6	5	4	3
9	0.93	0.07	0	0	0	0	0
8	0	0.87	0.13	0	0	0	0
7	0	0	0.83	0.17	0	0	0
6	0	0	0	0.76	0.24	0	0
5	0	0	0	0	0.77	0.23	0
4	0	0	0	0	0	0.85	0.15
3	0	0	0	0	0	0	1

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